Modeling of Planar Quasi-Tem Superconducting Transmission Lines

Dimitrios Antsos, Wilbert Chew, A. Landis Riley, Brian D. Hunt, Marc C. Foote, Louis J. Bajuk, Daniel L. Rascoe, and Thomas W. Cooley

Abstract—**Design oriented modeling of high-temperature superconducting thin-film microwave circuits is difficult when film thickness is of the order of the penetration depth of the fields. Involved formulas for loss, phase velocity and characteristic impedance [1] can be derived from the Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity. The parameters required by these formulas do not correspond to "readily measurable" observables that depend on the manufacturing process of the superconductor. In this paper an application of the phenomenological loss equivalence method [2] in modeling the microwave behavior of planar quasi-TEM superconducting transmission lines is presented. Measured and modeled *S*-parameters of an existing superconducting coplanar waveguide lowpass filter agree to within 0.3 dB in magnitude and 0.5 radians in phase. Extracted values for penetration depth and real part of the conductivity of the superconducting film are within 10% of the findings of other researchers.**

I. Introduction

Design and modeling of superconducting microwave circuits is a relatively new venture since the discovery of superconductors with transitions above the 77 K temperature of liquid nitrogen. When the simplifying assumption that film thickness is much greater than the penetration depth of the fields begins to fail, the complexity of the complete formulas for surface impedance [1], based on the BCS theory, makes it impractical to use this theory for prediction-based design of microwave circuits. Using these formulas, involved calculations, with many assumed values as design parameters, have to be repeated with every iteration process of the design. This process is repeated until the device exhibits the desired performance. In addition, due to the implicit and involved character of the BCS-theory based formulas, the designer has no intuitive feeling about how design parameters such as loss and characteristic impedance are affected by material parameters such as skin depth and normal conductance. The approach presented in this paper is an application of a phenomenological loss equivalence method [2] used in the context of a two-fluid theory [3] modeled superconductor. The mathematical results derived are validated by application thereof to an actual superconducting coplanar waveguide (CPW) microwave circuit [4].

II. Mathematical Model

The theoretical results derived are as follows. Due to the similarity between a normal conductor and a superconductor, the lossless *S*-parameters of two identical circuits, one made with a "perfect," zero-resistance conductor and the other with a superconductor, are the same (loss-free network). This means that one can model any known transmission line and calculate its electrical length and impedance from its geometrical dimensions using the standard formulas for normal conductors. This analogy breaks down when losses are included in the *S*-parameters. This is an important difference since the importance of high-temperature superconductors lies in reducing losses. The surface resistance of a normal conductor is proportional to the square root of the frequency. However the surface resistance of a superconductor is approximately proportional to the square of the frequency, in the limit where penetration depth is much smaller than film thickness [5].

A superconductor can be viewed as a normal conductor with a complex conductivity [3]. The real and imaginary parts of the conductivity correspond to the normal electrons and superconducting electron pairs (Cooper pairs), respectively, according to the two fluid theory. The conductivity can thus be expressed as [3]:

$$\sigma = \sigma_1 - j\sigma_2 = \sigma_n \left(\frac{T}{T_c}\right)^4 - j\,\frac{1 - \left(\dfrac{T}{T_c}\right)^4}{2\pi f \mu_0 \lambda^2}, \qquad T < T_c \tag{1}$$

where σ_n is the normal part of the conductivity (normal electrons), T is the absolute temperature, T_c is the critical temperature of the superconductor, f is the frequency, μ_0 is the permeability of free space and λ is the zero-temperature penetration depth of the magnetic and electric fields into the superconductor. Using this conductance one can calculate the additional internal impedance Z_i, in Ohms/meter, due to the penetration of the fields in the

---

Manuscript received May 14, 1991; revised December 16, 1991. The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

D. Antsos, W. Chew, A. L. Riley, B. D. Hunt, M. C. Foote are with the Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109.

L. J. Bajuk was with the Jet Propulsion Laboratory, California Institute of Technology. He is currently with the Department of Atmospheric Sciences, University of Washington AK-40, Seattle, WA 98195.

IEEE Log Number 9107449.

superconductor and the related surface impedance Z_s [2]:

$$Z_i = Z_s \cdot G \cdot \coth(\tau G \cdot A) \tag{2}$$

where

$$\tau = (1 + j) \cdot \sqrt{\pi \cdot f \cdot \mu_0 \cdot \sigma}$$

and

$$Z_s = \sqrt{j \cdot 2\pi \cdot f \cdot \frac{\mu_0}{\sigma}}.$$

Here G is the incremental inductance geometric factor and A is the cross-sectional area of the line under characterization. The attenuation α_c follows directly from the above [6]:

$$\alpha_c = \frac{\mathrm{Re}\,(Z_i)}{2\sqrt{\left(\frac{L}{C}\right)}} \tag{3}$$

in Nepers/m, where L and C are the distributed inductance and capacitance of the transmission line, respectively.

This series of calculations is easy to perform numerically with any mathematical CAD program, for every different set of values of the parameters, but gives no insight as to how each individual parameter affects the final result (the attenuation). The calculations are also impossible to enter into popular microwave CAD software packages such as Touchstone (TM) by EEsof Inc., which cannot handle complex algebra for circuit design and optimization.

The equations, however, can be reduced algebraically to obtain the following explicit formulas for Re (Z_i) and Z_0', the effective characteristic impedance, including the effects of loss:

$$\mathrm{Re}\,(Z_i) = \frac{B}{A \cdot \rho \cdot \psi} \cdot \left[\cos\left(\frac{\pi}{4} + \frac{\phi}{2} - \chi\right) + \exp(2B \cdot \cos\theta) \cdot \cos\left(2B \cdot \sin\theta + \frac{\pi}{4} + \frac{\phi}{2} - \chi\right)\right] \tag{4}$$

and

$$Z_0' = \sqrt{Z_0^2 - \frac{c \cdot Z_0}{2\pi \cdot \sqrt{\epsilon_{\mathrm{eff}}} \cdot f} \cdot \mathrm{Im}\,(Z_i)} \tag{5}$$

where

$$\mathrm{Im}\,(Z_i) = \frac{B}{A \cdot \rho \cdot \psi} \cdot \left[\sin\left(\frac{\pi}{4} + \frac{\phi}{2} - \chi\right) + \exp(2B \cdot \cos\theta) \cdot \sin\left(2B \cdot \sin\theta + \frac{\pi}{4} + \frac{\phi}{2} - \chi\right)\right]$$

$$\rho = |\sigma| = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$\phi = \arctan\left(\frac{\sigma_2}{\sigma_1}\right) - 2\pi$$

$$\theta = \frac{\pi}{4} - \frac{\phi}{2}$$

$$B = G \cdot A \cdot \sqrt{2\pi \cdot f \cdot \mu_0 \cdot \rho}$$

$$\psi = \sqrt{[\exp(2B \cdot \cos\theta) \cdot \cos(2B \cdot \sin\theta) - 1]^2 + [\exp(2B \cdot \cos \cdot \theta) \cdot \sin(2B \cdot \sin\theta)]^2}$$

and

$$\chi = \arctan\left(\frac{\exp(2B \cdot \cos\theta) \cdot \sin(2B \cdot \sin\theta)}{\exp(2B \cdot \cos\theta) \cdot \cos(2B \cdot \sin\theta) - 1}\right)$$

where Z_0 and ϵ_{eff} are the values of the characteristic impedance and effective dielectric constant calculated using circuit theory and c is the velocity of light in vacuum [7], [8]. From these, the effective wave slowing factor n arising from the extra inductance due to the penetration of the fields can be expressed as

$$n = \sqrt{1 + \frac{\mathrm{Im}\,(Z_i)}{2\pi \cdot f \cdot L}} \tag{6}$$

where

$$L = Z_0 \cdot \frac{\sqrt{\epsilon_{\mathrm{eff}}}}{c}$$

The corrected phase velocity (v_p') and the corrected effective dielectric constant $(\epsilon_{\mathrm{eff}}')$ can then be expressed as follows in terms of their original values:

$$v_p' = \frac{v_p}{n} \tag{7}$$

$$\epsilon_{\mathrm{eff}}' = \epsilon_{\mathrm{eff}} \cdot n^2. \tag{8}$$

These equations have been entered into MathCAD (TM) (MathSoft Inc.) and validated numerically. They were also entered into a Touchstone circuit file for modeling and design purposes.

III. APPLICATION AND VALIDATION OF THE MODEL

For validation of the model, data were used from measurements of an yttrium barium copper oxide (YBCO) superconducting thin-film coplanar-waveguide low-pass filter on a lanthanum aluminate substrate. The popular microwave CAD software package Touchstone was used for modeling. The YBCO filter was designed and produced at JPL for the Naval Research Laboratory (NRL) High Temperature Superconductivity Space Experiment (HTSSE) [4]. These films are of varying quality and properties and further characterization is desirable. Their performance in different environmental and electrical conditions needs to be evaluated. The two fundamental parameters, zero-temperature penetration depth and normal conductivity, were optimized for best fit between measured and modelled data and then compared to reported values from other sources. These extracted parameters were found to be in very good agreement (within 10%) with the findings of independent measurements.

The layout of the filter is shown in Fig. 1. It consists of alternating high and low impedance lines with ground plane on both sides (coplanar waveguide (CPW)). The input and the output are K-connector-to-CPW transitions and the taper has been optimized to hold characteristic impedance to a constant 50 Ω. The filter was modelled in Touchstone using generic transmission line elements including loss calculated using (3).

The long equations (4) and (5) were broken into smaller sub-equations so as to accommodate the limited capabilities of the program. The impedance steps were modeled using 2-port S-parameter files (s2p files). These were calculated using an electromagnetic analysis software package which uses the Galerkin method of moments for analyzing an arbitrary geometry circuit for its S-parameters [9]. The software package is *EM* by Sonnet Software Inc. The S-parameters were de-embedded with reference planes at the discontinuity. Thus they do not contribute additional phase or loss. A similar analysis was performed for the input and output 50 Ω tapers and they are included as S-parameter files as well. The curved geometry of the circuit is ignored in the analysis (i.e., the transmission lines are assumed to be of the same length on a straight line). Dielectric losses were considered in the analysis though they turn out to be orders of magnitude smaller than resistive losses in our case and can actually be neglected. They are included, however, for the sake of completeness of the model.

The dielectric loss tangent of the lanthanum aluminate ($LaAlO_3$) substrate was optimized for best fit between measured and modeled data. The optimum value for it was 0.0001. If set to zero, it would affect the final modelled S21 curves by less than 0.05 dB. The two main parameters which were optimized in the final run for loss match at the liquid nitrogen temperature of 77 K are the zero-temperature penetration depth λ and the normal conductivity component σ_n of the superconductor. The lengths of the input and output tapers and the 50 Ω lines were optimized for phase match.

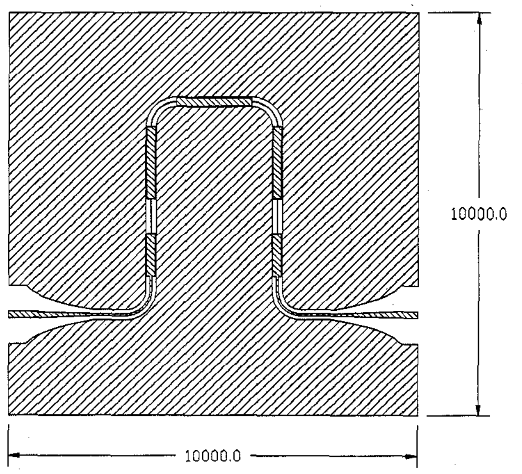


Fig. 1. Mask of the YBCO filter (dimensions in microns).

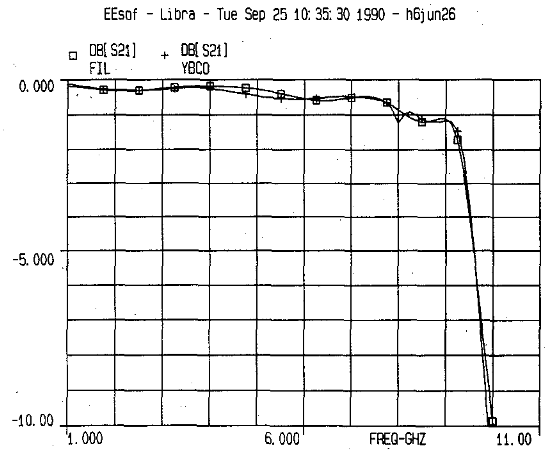


Fig. 2. Measured (YBCO) versus modeled (FIL) magnitude of S21.

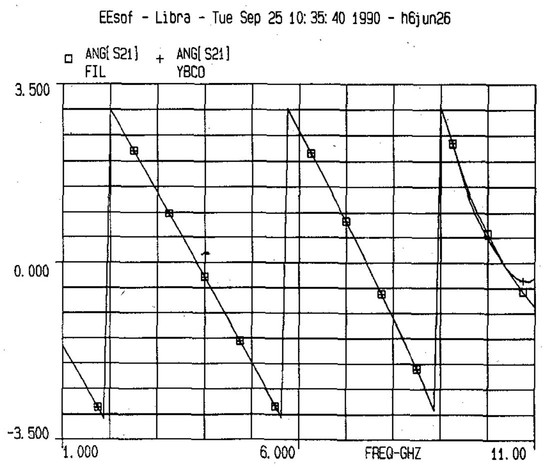


Fig. 3. Measured (YBCO) versus modeled (FIL) phase of S21.

Figs. 2 and 3 are plots of measured versus modeled values of magnitude and phase of S21, respectively. The agreement between modeled and actual data is very good. The almost perfect match of the slopes of the phases indicates that the formulas successfully account for wave slowing due to any additional penetration-depth-induced inductance. The extracted optimized parameters are as follows:

$$\lambda = 4830 \text{ Å}$$

$$\sigma_n = 1.8 \times 10^6 \text{ S/m}.$$

All measurements which were used for modeling-fitting purposes were made at liquid nitrogen (LN_2) temperature of 77 K. The critical temperature of the films, from previous dc measurements, were found to be in the range

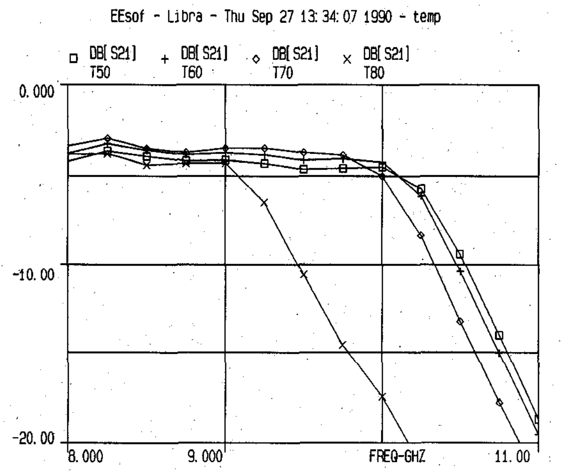


Fig. 4. Measured (YBCO) response at 50, 60, 70, and 80 K.

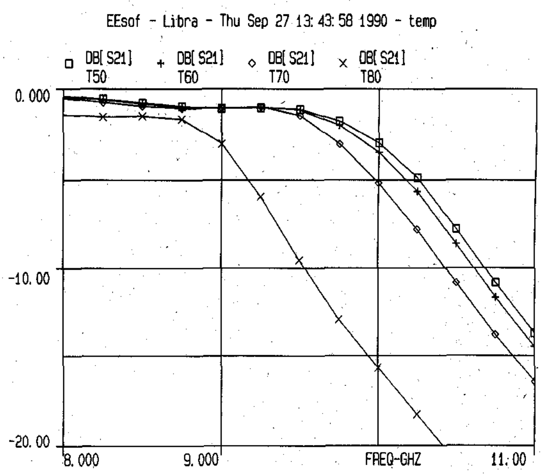


Fig. 5. Modeled response at 50, 60, 70, and 80 K.

from 83 to 88 K. For modeling purposes, a T_c of 85 K was assumed since this parameter could not be optimized as it is not linearly independent of σ_1 and σ_2. The inaccuracy in that assumption is about an 8% difference in λ per degree of difference of T_c from its actual value.

The values obtained for the parameters λ and σ_n are reasonable values. Polakos *et al.* from AT&T Bell Labs report on a similarly deposited high-temperature superconducting microstrip circuit with a zero-temperature penetration depth of 4500 Å, very close to the value extracted from this analysis [10]. As they point out in their paper, this is only an overall weighted average of the real penetration depth, the latter being larger in non-uniform areas (corners, imperfections) of the film. Published data on state of the art high-purity crystals give values of 1400 Å for zero-temperature penetration depth and 1.14×10^6 S/m for normal conductivity [11]. The discrepancy between the single crystal and film results may be due to the existence of grain boundaries or defects in the YBCO film.

Further validation for the extracted values and the proposed model is obtained from the temperature-dependence agreement between measured and modeled data. The only data which was calibrated correctly (de-embedded to the reference ports of the circuit) is that from the measurements made at 77 K with the circuit immersed in LN_2. The rest of the measurements, taken at different temperatures in the range from 15 to 95 K, were performed in the vacuum jacket of a closed-cycle refrigerator. The connectors in the refrigerator could not be calibrated out and their loss and phase is included in the measured S-parameters. These losses, assumed approximately constant over the frequency range from 7 to 11 GHz, translate the actual S21 curves up or down in the graph but do not affect the slope of $|S21|$ away from the pass-band. Fig. 5 was obtained by assuming the model derived from the 77 K data valid and merely changing the parameter T to the measured values. It shows the predicted filter response at various temperatures. The good agreement of the predicted roll-off beyond the corner frequency with the measured curves plotted in Fig. 4 indicates that the validity of the model covers the whole range from 50 K to about 80 K. Further modeling reveals satisfactory results in the 15 to 95 K temperature range.

IV. CONCLUSION

A practical mathematical model for use in design-based modeling of high-temperature superconductor microwave circuits has been presented. Application of the model to an existing circuit exhibits very good agreement between the model and the actual circuit and suggests the existence of imperfections in the modeled crystalline superconducting circuit. Given pre-measured material parameters, namely the normal conductivity and the zero-temperature penetration depth of the superconductor, complete modeling of arbitrary planar TEM high-temperature superconductor circuits is now possible using commercially available conventional microwave CAD software packages.

V. REFERENCES

[1] P. B. Miller, "Surface impedance of superconductors," *Phys. Rev.*, vol. 118, pp. 928–934, 1960.

[2] H. Lee and T. Itoh, "Phenomenological loss equivalence method for planar quasi-TEM transmission lines with a thin normal conductor or superconductor," *IEEE Transactions Microwave Theory Tech.*, vol. 37, pp. 1904–1909, 1989.

[3] T. Van Duzer and C. W. Turner, *Principles of Superconductive Devices and Circuits.* New York: Elsevier, 1981.

[4] W. Chew, A. L. Riley, B. D. Hunt, M. C. Foote, L. J. Bajuk, and D. L. Rascoe, "Design and performance of a high T_c superconductor coplanar waveguide filter," *IEEE Transactions Microwave Theory Tech.*, Special Issue on Microwave Applications of Superconductivity, vol. 39, pp. 1455–1461, 1991.

[5] Superconductor Technologies, "Surface resistance of thin films," *Testing Service Technical Bulletin*, Santa Barbara, CA, 1988.

[6] S. Ramo, J. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics.* New York: Wiley, 1984.

[7] K. Gupta, R. Garg, and I. Bahl, *Microstrip Lines and Slotlines.* Dedham, MA: Artech, 1979.

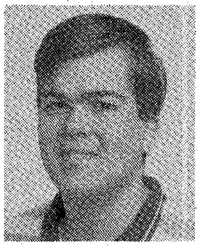
[8] G. Ghione and C. Naldi, "Parameters of coplanar waveguides with lower ground plane," *Electron. Lett.*, vol. 19, pp. 179–181, 1983.

[9] J. C. Rautio and R. F. Harrington, "An electromagnetic time-harmonic analysis of shielded microstrip circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 726–730, 1987.

[10] P. A. Polakos, C. E. Rice, M. V. Schneider, and R. Trambarulo, "Electrical characteristics of thin-film $Ba_2YCu_3O_7$ superconducting ring-resonators," submitted to *IEEE Trans. Microwave Guided Wave Lett.*

[11] D. R. Harshman *et al.*, "Magnetic penetration depth in single crystal $YBa_2Cu_3O_7$," *Phys. Rev.*, vol. B 39, p. 2596, 1989.

[12] K. B. Bhasin, C. M. Chorey, J. D. Warner, R. R. Romanofsky, V. O. Heinen, K. S. Kong, H. Y. Lee, and T. Itoh, "Performance and modeling of superconducting ring resonators at millimeter-wave frequencies," in *1990 IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 269–272.

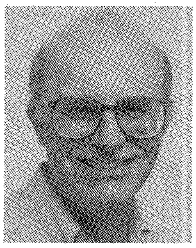


Dimitrios Antsos received the B.S. and M.S. degrees in electrical engineering from the California Institute of Technology (Caltech) in 1990 and 1991, respectively, and is currently working on the Ph.D. (expected June 1993) at the same school. His Ph.D. research is on microwave applications of high-temperature superconductors.

He is a Member of Technical Staff of the Spacecraft RF Telecommunications Group at the Jet Propulsion Laboratory. Since first joining the group in May 1989, he has worked on various research projects, including computer aided design and modeling of microwave circuits and array antennas and system phase noise analyses.

Wilbert Chew received the Ph.D. from UCLA, Los Angeles, in 1988, where he developed a millimeter-wave integrated quasi-optical FET mixer and imaging and ranging demonstrations with a small array of such mixers.

From 1981 to 1983, he was a of the Technical Staff at Hughes Aircraft Company, doing modeling and systems analysis of infrared imaging systems. In 1989, he joined the Jet Propulsion Laboratory, California Institute of Technology, where he has participated in antenna array analysis and development of microwave applications for high-temperature superconductors.



A. Landis Riley obtained the Ph.D. in electrical engineering in 1971 from Johns Hopkins University.

He is Manager of NASA Communications Technology Programs at the Jet Propulsion Laboratory. He has over 20 years experience in the design and development of microwave and millimeter wave components, subsystems, and instruments. His experience includes four years at Westinghouse Electric Corporation designing active microwave components, four years at MIT Lincoln Laboratory developing spaceborne frequency synthesizer microwave subsystems, and twelve years at JPL involved in component and subsystem design at frequencies as high as 230 GHz. This work has included development of a 55 GHz spaceborne mixer, system development of the Microwave Limb Sounder instrument, and development of 115 GHz and 230 GHz radiometer system for remote sensing for the Martian atmosphere. He has been responsible for the design and development of microwave and millimeter wave solid state power amplifier systems. He managed the development of the JPL device for the Naval Research Laboratory HTSSE program.

Brian D. Hunt received the Ph.D. in Applied Physics from Cornell University in 1984. His thesis research involved a study of three terminal superconducting devices and nonequilibrium superconductivity utilizing a novel double edge junction structure fabricated by electron beam lithography.

He is currently a member of the Technical Staff in the Microdevices Laboratory at the Jet Propulsion Laboratory, California Institute of Technology. He worked for three years at the General Electric Corporate Research and Development Laboratory where he investigated the application of Si molecular beam epitaxy to the growth of epitaxial silicide/Si heterostructures and devices including a $Si/CoSi_2/Si$ metal base transistor. Since arriving at the Jet Propulsion Laboratory in 1987, he has initiated the high temperature superconductor thin film devices program at JPL. His work at JPL has focused on epitaxial growth studies of YBaCuO by laser ablation, as well as high Tc device studies utilizing superconductor/normal-metal/superconductor (SNS) structures to optimize the surface electrical properties of YBaCuO. He invented and fabricated a novel edge-geometry YBaCuO/Au/Nb device structure, as well as the first NbN/MgO/NbN edge-geometry tunnel junctions. He has also been a collaborator on a successful project to fabricate and deliver flight-qualified, YBaCuO microwave filters for a satellite lunch. Recently, he fabricated the first YBaCuO/normal-metal/YBaCuO edge geometry weak links utilizing nonsuperconducting Y-Ba-Cu-O barrier layers.

In 1991, Dr. Hunt was awarded the Lew Allen Award for Excellence. He has published over fifty scientific papers: recent representative papers can be found in *Appl. Phys. Lett.* 53, 2692 (1989); *Appl. Phys. Lett.* 56, 2678 (1990); *and Appl. Phys. Lett.* 59, (8/19/91).

Marc C. Foote received the Ph.D. in low-temperature physics from the University of Illinois, Urbana-Champaign. The work for his thesis involved electrical properties of disordered materials, and was performed at AT&T Bell Laboratories under a Ph.D. scholarship program.

He is a member of the Technical Staff in the Microdevices Laboratory at the Jet Propulsion Laboratory, California Institute of Technology. His current work involves developing $YBa_2Cu_3O_{7-x}$ thin film growth techniques as well as fabricating superconductor-normal metal-superconductor devices using this material.

Louis J. Bajuk received the B.A. in physics in 1989 at Oberlin College, where he helped initiate a high temperature superconductor research program.

He is a member of the Nanodevices and Superconductivity Group of the Microdevices Laboratory at the Jet Propulsion Laboratory (JPL), California Institute of Technology, where he has worked on the deposition of laser-ablated high temperature superconductor thin films, as well as the fabrication of YBCO thin film passive microwave devices. He is currently on leave of absence from JPL and is a graduate student with the Department of Atmospheric Sciences, University of Washington.

Daniel Lee Rascoe received the Ph.D. in physics in 1978 from the University of Illinois where his research involved state of the art low temperature refrigeration, mm-wave (70–90 GHz) electron spin resonance measurements, and observations and analysis of phonon transport through superfluid Helium. His research studies also included the evaluation of several piezoelectric and optical materials at low temperatures and microwave frequencies.

He has over 13 years experience in the development of microwave and mm-wave spacecraft components and subsystems. This experience includes eight years at Hughes Aircraft Space and Communications Group where he designed and developed flight communications receivers, low noise amplifiers, and solid state power amplifiers operating from 2–60 GHz. For the past five years, he has been the cognizant engineer at JPL for the development of spacecraft Ka-band (32 GHz) phased array technology which incorporates monolithic microwave integrated circuits (MMIC's) and is presently introducing fiber optic technology for signal distribution and electronic beam steering control. He is presently the Technical Group Supervisor of the Spacecraft RF Development Group in Section 336 where his groups tasks include several mm-wave array development programs, a Ka-band telecommunications and science subsystem for the Cassini spacecraft, and the development of high temperature superconducting microwave components for spacecraft communications.

Thomas Cooley received the Bachelor's degree in electrical engineering from Rensselaer Polytechnic Institute.

He then joined JPL's Microwave Communications Group in 1988. While he continued working at JPL, he attended Caltech and received the Master's degree in electrical engineering in 1991. He is currently on a leave of absence from JPL attending the Univerity of Arizona, in pursuit of the Ph.D. in electrical engineering and optical science under a NASA Fellowship.